Low Energy Nuclear Reactions (LENR) - and Nuclear Transmutations at Unitary Quantum Theory

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Abstract

In this article is discussed problems Low Energy Nuclear Reactions – with position unitary quantum theory. Probability of these phenomena more than predicts usual quantum theory for small energy.

Keywords: Unitary Quantum Theory, Cold Nuclear Fusion, Low Energy Nuclear Reactions, Coulomb Repulsion, Quantum Mechanics, Coulomb barrier, Nuclear Transmutation

1. Introduction

“...The kernels are pure emeralds, but people, it may be, lie...”

A.S. Pushkin

Let us to analyze the epoch-making experiments carried out by M.Fleishman and S.Pons in the March of 1989 [1] and revealed for the first time the phenomenon called the cold nuclear fusion (or Low Energy Nuclear Reactions-LENR), i.e. the nuclear synthesis at low temperature. Notice, one of the authors of this article (prof. L.Sapogin) has predicted already in 1983 [2] in his works the possibility of such nuclear reactions at small energies. Without going into well-known details we can say: the phenomenon of the cold nuclear fusion really exists and no one physicist can explain it clearly within the classical mechanics or within the standard quantum mechanics. The series of various mechanisms which explain this intriguing phenomenon is offered but it is hard to believe them because of the following reasons.

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The curve of nuclear potential energy in the case of a charged particle interaction with a nucleus is plotted in Fig.1, where the right top part of the curve corresponds to the mutual Coulomb repulsion that nucleus and charged particle is experienced.

The repulsion potential is described by formula

$$U(r) = \frac{ZZe^2}{r},$$

where $Z$ is the nucleus charge, $z$ is the charge of particle moving to the nucleus, $e$ is the electron charge; $r$ is the distance between given particle and nucleus. At $r=R$ the potential energy curve has a jump that can be explained by the appearance of the intensive nuclear attraction. Nowadays, we do not know any mathematical formula for the potential of the nuclear attraction. If the charged particle is able to overcome the potential barrier of the height

$$B_c = \frac{ZZe^2}{R} \approx \frac{Zz}{\sqrt[3]{A}} \text{MeV},$$

then further the particle falls into the region of nuclear forces of attraction and the nuclear reaction will proceed.

Let us consider the nuclear interaction if the charged particle possesses kinetic energy $T < B_c$. From the classical mechanics point of view there will no nuclear reaction at all in that case because reaching some distance $r < R$ to the Coulomb barrier top the particle will be turned back and reflected. Deuteron energy in ordinary electrolytic cell of Fleishman-Pons is near $0.025$ eV, the height of Coulomb barrier in this case is $B_c = \frac{ZZe^2}{\sqrt[3]{A}} = 0.8 \text{MeV}$. It is naïve to discuss the question about overcoming the barrier with the height dozens of million times more than the kinetic energy from the classical mechanics point of view.

However, from quantum mechanics point of view there is tunneling effect and the probability of such tunneling, or potential barrier transparency $D$, is given by well-known formula:

$$D \approx \exp \left( -\frac{2}{\hbar} \sqrt{2\mu(U - T)} dr \right)$$

Fig.1. Potential corresponding to nuclear fusion.
where \( \mu = \frac{Mm}{M + m} \) is so called reduced mass, \( M \) is the nucleus mass, \( m \) is the particle mass. The lower limit of integration \( r_1 \) coincides with nucleus radius \( R \), the upper limit \( r_2 \) corresponds to condition \( T = \frac{Zze^2}{r_2} \). After integrating we will obtain

\[
D = \exp(-2g\gamma)
\]

where \( g = \frac{R}{\lambda_{B_c}} \); \( \gamma = \sqrt{\frac{B_c}{T}} \arccos\left( \frac{T}{B_c} \right) - \sqrt{1 - \frac{T}{B_c}} \), and value \( \lambda_{B_c} = \frac{\hbar}{\sqrt{2mB_c}} \), is de Broglie wavelength, corresponding to the particle kinetic energy equal to the barrier height \( T = B_e \). If \( T << B_e \), then formula (1) can be easily transformed into the form

\[
D = \exp\left(-\frac{2\pi RB_c}{\hbar v}\right) = \exp\left(-\frac{2\pi Zze^2}{\hbar v}\right)
\]

where \( v \) is velocity.

If we estimate the values \( g \) and \( \gamma \) for collision of two neutrons with such energy, then we obtain following:

\[
g = \frac{R\sqrt{2mB_c}}{\hbar} = 1.9; \quad \gamma = \sqrt{\frac{B_c}{T}} \arccos\left( \frac{T}{B_c} \right) - \sqrt{1 - \frac{T}{B_c}} \approx 8883 ,
\]

hence the probability of such a process equals to \( \exp(-2 \cdot 1.9 \cdot 8883) \approx 10^{-7328} \) (!). The cross-section of fusion reaction can be determined as multiplication nuclear cross-section and tunneling probability, i.e.

\[
\sigma = \sigma_{\text{nuc}}D.
\]

Moreover, if the deuteron sighting parameter does not equal zero, then the appearance of centrifugal potential

\[
U = \frac{\hbar^2 l(l+1)}{2mr^2}
\]

will lead for more reducing of interaction probability.

2. Experimental results.

The obtained values do not require a commentary. It is quite explainable that the official physical science has rejected every talks about the possibility of the LENR. The experiments of M.Fleishman and S.Pons were declared as some misunderstanding. For example, the most serious and responsible edition Encyclopedia Britannica 2001 could not even find a place for the cold nuclear fusion concept. Such official viewpoint can be understood only if one considers standard quantum mechanics as absolutely valid. In spite of all during last 14 years starting from the moment of experimental discovery of M.Fleyshman and S.Pons about 50 international conferences dealing with that subject were organized, there are a lot of books, Journals, and magazines discussing this problem, the number of articles written about it is near to dozen of thousand. Today the situation is changing step by step into positive direction. And the researches are slowly turning away from the high road of hot fusion that have wasted during last 60 years more than 90 billion dollars for nothing.
The LERN experimental data are extremely numerous and various, but we are going to dwell on the most important and fixed results. Thus at classical electrolysis study of the palladium cathode saturated with deuterium there is enormously great heat generation in heavy water: up to 3-kilowatt/cm³ or up to 200 megawatt-second in a small sample. There were also detected fusion products: tritium \((10^{-7} - 10^5 \text{t/sec})\), neutrons with the energy equal to 2.5 MeV \((10-100\text{n/sec})\), helium. The absence in the products of the reaction \(He^3\) shows that heat does not the result from the reaction \(d+p\). More over one can observe the emanation of charged particles \((p, d, t, \gamma)\). We can study similar processes at gas discharge over palladium cathode, at change of phase in various crystals saturated with deuterium, at radiation treatment of deuterium mixture by strong sonic or ultrasonic flux, in cavitations micro-bubbles in heavy water, in a tube with palladium powder saturated with deuterium under the pressure of 10-15 standard atmospheres and others. In some reactions, (for example at \(d + t \rightarrow \alpha + p\) ) neutrons with the energy 14 MeV are absent; one can meet the same strange situation in other cases too. Thus the participation of nucleus \(Li^6, Li^7\) in reactions with deuterium and protons, while the reaction

\[
K^{39} + p \rightarrow Ca^{40}
\]

was fixed even in biological objects. But the most intriguing fact in all these processes is the lack of fusion products that could explain the calorific effects. Thus, in some cases the number of fusion products (tritium, helium, neutrons, and quantum) should be million times more to give any explanation of the quantity of the heat evolved. So great energy liberation can be explained neither by chemical or nuclear reactions nor by changes of phase. More details about the magic source of such energy are given in the books [3, 4, 21]

The deeply studied interaction \(d+d\) proceeds along three channels:

\[
\begin{align*}
D + D &\rightarrow T(1.01) + p(3.03) \quad \text{(1 channel)} \\
D + D &\rightarrow He (0.82) + n(2.45) \quad \text{(2 channel)} \\
D + D &\rightarrow He + \gamma (5.5) \quad \text{(3 channel)}
\end{align*}
\]

These reactions are exothermic. The third channel has very low probability. In the result of experiments it have been discovered that these reactions can take place at indefinitely small values of energies. In molecule of \(D_2\) the equilibrium distance between atoms is 0.74Å and according to standard quantum theory these two deuterons would be able to come into nuclear fusion by chance. But the value of the interaction is quite small [5] and equals \(\lambda_{D_2} = 10^{-64}\text{c}^{-1}\). There is an estimate well known in literature [5]: the water of all seas and oceans contain \(10^{43}\) deuterons and there would be only one fusion within \(10^{14}\) years.

It is evident from the sated above that the main obstacle preventing \(d+d\) reaction is the presence of an extremely high Coulomb barrier. The approach given in the [3, 4, 21] allows to solve that problem. The UQT also gives such possibility. Solutions of some UQT equations show that distance the deuterons could draw close depend strongly on the phase of wave function (by the way that is absolutely evident by intuition).

3. LERN and Nuclear Transmutations at the Unitary Quantum Theory.

Let us consider the motion of a charged particle to the nucleus from the viewpoint of UQT using the equation with oscillating charge in one-dimensional case [3, 4, 6-11, 21]. Assume there is an immovable nucleus with the charge \(Ze\) placed in origin \(x = 0\), and the particle with the charge \(Ze\), and mass \(m\) is moving towards this nucleus with some initial velocity along axis \(x\).
Autonomous and non-autonomous equations of the particle motion were deriving from Schrodinger equation for very small kinetic energy [3,4,6-11,21] and have the following form for Coulomb potential:

\[
m\frac{d^2 x}{dt^2} = \frac{2Ze^2}{x^2} \cos^2\left(\frac{m}{\hbar} \frac{dx}{dt} x + \phi_0\right),
\]

(3)

\[
m\frac{d^2 x}{dt^2} = \frac{2Ze^2}{x^2} \cos^2\left(\frac{m}{2\hbar} \left(\frac{dx}{dt}\right)^2 t - \frac{m}{\hbar} \frac{dx}{dt} x + \phi_0\right),
\]

(4)

where \(\phi_0\) is the initial phase. These equations were numerically integrated under following starting data: \(Z=z=1, e=1, m=1, x_0 = -10, \hbar = 1\) and different initial velocities and initial phases. As it were expected, the particle’s braking and acceleration took place in the moments the oscillating charge is big. But at the final stage at some initial phases close to \(\frac{\pi}{2}\) a delightful process appeared. The velocity, charge and repulsive force are very small. Due to the phase relationship small charge stay constant during long period, and that means that nothing affects particle (or, rather, its remainder), the particle very long snails with low and constant velocity inside the other particle field (“snail effect”) and may approach its center at close distance. That process bears a strong resemblance to slow inconspicuous spy penetration into the hostile camp.

**Fig.2:** Distance to the turning point of moving charge in respect to value of initial phase for different velocities.

That phenomenon appears within some area of phases and is convenient to call it a phase hole, which is illustrated by plots in Fig.2 (obtained after integration of the equation (4)). Besides, it may be possible now to explain one of the anomalies of the nuclear physics (which as if does not exist according to physics literature). When the nucleon energy equals 1 MeV its velocity equals \(10^9\) cm./sec., nucleus radius equal to \(10^{-12}\) cm., the time of flight through nucleus equals \(10^{-21}\) sec., but time interval within which the nucleon flies out is usually anomalous huge - \(10^{-14}\) sec, it is even out of understanding what does the nucleon do inside the nucleus for such a long time? But it can be easily explained in the frame of our theory by “snail effect”.

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That phenomenon is studied more detailed in books [3,4] and Section 4.

![Graph](image)

**Fig.3: Minimal distance between charges in respect to initial velocity for different values of initial phase**

For the same equation, the minimal distance between charges was computed depending as a function of the velocity and at various values of the initial phase. For comparison, the result of classical computation on the base of Coulomb law is shown in Fig.2. We can see from the same plots that the minimal distance at which the particle is able to come near the nucleus does not depend on the kinetic energy. But with the decrease of velocity the width of initial phase interval corresponding to minimal distance decreases too. In other words with decrease of energy the probability of nuclear fusion also decreases. We obtain on the whole the same results for autonomous equation (3).

In accordance with the standard quantum theory the relation of fusion velocity along tritium and neutron channels should be near unity: \( \frac{v}{n} \approx 1 \). But the results of numerous experiments of the cold fusion show that value greatly differs from unity and equals to \( \frac{v}{n} \approx 10^9 \). That value is reproduced in different experimental situations and by various experimental groups with a high accuracy. Till now that quite intriguing problem have not been solved. We will try to explain the possible reason for that. Neutrons are influenced at low velocity within the phase hole by forces of nuclear attraction and protons - by the forces of electrostatic repulsion. Under the influence of momentum of given forces the deuteron had enough time to turn in such a way that its neutron parts are turned to each other. After the neutrons attraction the saturation of nuclear forces appears. That weakens the connection between protons and one of them is able to leave the system. Schematically, the reaction may be rewritten in the form

\[
d + d \rightarrow p + (n + d) \rightarrow p + t
\]

That looks like effect of Oppenheimer-Phillips [12].

But it is precisely known that at high energies the probabilities of the first and the second channels of the reaction are similar and that phenomenon should be anyhow explained. The growth of the probability of neutron channel with the increase of the energy may deal with the secondary neutrons birth in reaction \( T + D = He + n (14.1 \text{ MeV}) \).
In medium full of heavy hydrogen the most part of being born tritons will transform into neutrons due to that reaction. The cross-section of this reaction is equal to 5 barn at energy of 70 KeV. In accordance with the estimate in [12], the numbers of so secondary neutrons for one triton are $7.9 \cdot 10^{-12}, 1.7 \cdot 10^{-9}, 2.7 \cdot 10^{-6}$ for the triton energies 10, 20 and 100 KeV correspondingly. Thus the prevalence $\frac{t}{n} > 10^6$ must be expected in those reactions only, where the birth of tritium corresponds to energies higher than 40 KeV [12].

We should not think that phenomenon of phase hole will result in nuclear reaction over the whole area of the hole. We can assume that along with decrease of Coulomb repulsion value, the value of the strong interaction decreases too. How? Today nobody knows the exact equation for strong interaction potential. Furthermore the particle reaches turning points $x_{\text{min}}$ “losing flesh (charge) enough”. Will the particle be able to participate in an honest nuclear reaction or just pass it through as an electron in s-states of atom does? But there are very narrow phase areas where shortly after the particle stops its charge is rapidly growing and particle velocity increases abruptly. The charge may be even maximal within the scope of nuclear forces. Apparently this narrow area is responsible for the cold nuclear fusion. And probably at strong interactions the phase hole is working too.

It was discovered long ago that nuclear transmutations are wide spread (it is especially evident for plants and biological objects), but they are faintly connected with energy liberation. The examples of such reactions are:

$$Mn^{55} + p \rightarrow Fe^{56}$$
$$Al^{27} + p \rightarrow Si^{28}$$
$$P^{31} + p \rightarrow S^{32}$$
$$K^{39} + p \rightarrow Ca^{40}$$

In reactions of such a type very slow proton (its kinetic energy is equal practically to zero) is penetrating inside the nucleus by the above-mentioned way and stays there. There is no nuclear energy liberation, because the nucleus remains stable both before and after reaction. In accordance with classical nuclear physics, the nucleus, as usual, after a charged proton with great kinetic energy gets inside it, becomes unstable and breaks to pieces, and its fragments obtain bigger kinetic energy. The reactions of above-mentioned type were considered impossible at all at small energies and therefore were not studied in the classical nuclear physics. Apparently, that is absolutely new type of nuclear transmutations unacknowledged by modern nuclear science, but experimentally discovered sufficiently long ago. Today there are a lot of experimental data confirming the mass character of nuclear transmutation. Moreover there are many projects of nuclear waste neutralization that use this method. The journals “Infinite Energy”, “New Energy”, “Cold Fusion”, “Fusion Facts” etc. and Internet is full of such projects.

Of course, if the charge of a nucleus changes, then the electron shells of atom also will re-form, but the energy dealing with that process will be of few electron-volts order and cannot be compared with in any case with the energies of nuclear reactions that are from units till hundreds of billions electron-volts. By the way, experts in nucleonic got used to that range energies in nuclear reactions. Exactly that circumstance forces them it to reject a priori the presence of any nuclear processes in biology, because at such debris’ energies dozens and hundreds of thousands of complex biological molecules will be destroyed.

Quite far ago Louis C. Kervran [13] has published the book about nuclear transmutations in biology, and now nearly 20 years after it was reissued! Apparently for the first time numerous experimental data describing the above-mentioned phenomena were presented.
The reaction of official science was also quite interesting. For example, the well-known physician Carl Sagan after having read the book about experimental results advised Kervran to read an elementary course of nuclear physics!

A little bit later Panos T. Pappas [14] researched one of the nuclear reaction perfectly observed within biological cells, viz.

\[ Na_{23}^{11} + O_{16}^{8} = K_{39}^{19} \]

The existence of \( K - Na \) balance is well known in the classical biology for the long time. The ratio between quantities of \( K \) and \( Na \) ions is kept with a great accuracy in spite of presence of any \( K \) or \( Na \) ion in the food. Later in the work [15] that nuclear reaction was called “equation of life” and M. Sue Benford proved with direct physical methods the presence of such nuclear reactions in biological objects. To our regret there are too few researches of those problems in biology. We know about the existence of such groups in Japan (Komaki), India and Russia.

All programs of controlled nuclear fusion are based on meaningless heating and pressing of the respondent material. In spite of successes achieved, the head of such a group in England Dr. Alan Hibson (private communication) announces few years ago that not less than 50 should pass before the construction of reactor for demonstration can be ready. Today that point of view becomes generally accepted. Note that the reactor itself, even if it were constructed in future (the authors greatly doubt that possibility) would be extremely complicated, expensive and environmentally pollutant.

Classical approaches have not achieved positive results yet in spite of investments of many billions and huge number of physicists, engineers, maintenance staff, managers and chief-managers involved. Of course that enormous army of researches became a potential enemy of any alternative projects of fusion. It was noted that “vitality” of any idea is directly proportional to the amount of persons involved and capital invested. Those were the reasons why works of M. Fleishman and S. Pons were given a hostile reception.

Each program of controlled nuclear fusion has adjective “controlled”, but as a matter of fact there is no control at all. The initial quantity of respondent material is simply very small, quite providently we should say. For example a ball of lithium deuteride used for laser reduction is near 1-2mm in diameter. But nobody has at least examined the question of energy recovery to be generated in the result of that ball explosion. By the way the energy from that explosion is nearly equal to energy obtained in the result of an anti-tank grenade explosion.

Straightforward approach to nuclear fusion used by modern science is absolutely natural because there is no method in the standard quantum mechanics to influence that process. The future of systems of really controlled nuclear fusion will possibly lie not on the path of the primitive and meaningless heating and pressing of the respondent material but on the path leading to the collision of nuclei possessing a small charge and micro adjusted wave function phase.

That is possible in principle by the superposition of controlling external electromagnetic field on the reactive system containing quasi-fixed order atoms of deuterium and free deuterons. The special atomic lattice geometry may produce the same characteristics. Dispersion of a deuterons flow due to diffraction on such lattice will result in automatic selection of deuterons in energies and phases.

Apparently in electrochemical experiments carried out by M. Fleishman and S. Pons, such ordered system existed inside the \( Pd-D \) lattice and as the result appeared weak phasing able to explain the results of experiments raised [18,19].

We suppose that in future models of the reactors in contrast to all existing projects will react in any moment of time only the smallest part of deuterons automatically selected relative to initial phases.
It could be possible to obtain in result the small energy generating during long period of time until the reserve of light reacting nuclei will not be exhausted. That fusion does really have the right to be called “controlled”.

Today we can imagine that in the future the processes of cold fusion will be adopted probably not in energy production but for atomic wastes utilization and isotopes manufacture.

Many experimentalists [18,19] discovered that the quantity of the heat generated in the common water electrolysis over nickel electrodes (in that system we cannot even expect nuclear fusion presence) were the same as in the case with electrolytic lattice with heavy water. That fact confirms the results of other experiments in the process of which it was discovered that the number of fusion products was in millions times less than it was necessary for that quantity of generated heat, and its origin was mysterious. We had examined the question of heat origin in books [3.4, 21].

The thermal cell CETI (created by James Patterson in 1995 [20]), where is going on an electrolyze of specially manufactured nickel bolls in common water, has shocked scientists in USA. American newspaper «Fortean Times» No 85, 1995, wrote about it: “December the 4th, 1995 will go down to history!”. At that day the group of independent experts from five American Universities tested the work of new energy source with stable output heat rating 1.3 kWatt. The electric energy input was 960 times less.” All experts noted that generated heat had enigmatic origin and would not be explained neither by chemical or nuclear reactions nor by phase transitions. By American ABC TV there were two telecasts at 7th and 8th of February, 1996 in cycles «Nightline» and «Good Morning America» about Patterson creation of new source of energy, able to generate in hundred times more energy than it had consumed. And again it were accentuated that the origin of generated heat remains mysterious. It is interesting that American Company Motorola made attempts to buy the patent for cell CETI for US$ 20.000.000, but was rejected. We are sure that Motorola Company had spent a certain sum for the study of that problem before making so serious an offer. All processes within the Patterson cell do not concern nuclear reactions (although Patterson thinks otherwise), and at our opinion can be explained with the same processes used here above [3, 4, 21] for the description of proton-conductive ceramics.


Further we will give certain concrete data demonstrating the phase values of a deuteron with an oscillating charge, under which the deuteron can approach the nucleus to a critical distance of \(10^{-12} \text{ cm}\) or less, i.e. giving the data for estimating the value of the above-mentioned phase hole in the interval \((0, \pi)\) of the phase change.

Assume that the stationary nucleus with the charge \(q\) is placed at the coordinate origin \(x=0\) and a deuteron with the same charge \(q\) is placed at the initial moment \(t=0\) at the point \(x_0 < 0\) on the \(x\)-axis, and the deuteron velocity equals \(\dot{x}_0 = v_0 > 0\). The units of mass, length and time are chosen in such a way that \(m = 1, \hbar = 1, c = 1\) (\(m\) – deuteron mass, \(c\) – light velocity). Charge \(q\) equals \(0.085137266\). Our units are connected (up to 4 significant figures) with the system (kg, m, s) as follows:

- 1 mass unit = \(3.345 \cdot 10^{-27}\) kg,
- 1 length unit = \(1.049 \cdot 10^{-16}\) m,
- 1 time unit = \(3.502 \cdot 10^{-25}\) s.

The electron velocity corresponding to its energy of 1 \(\text{eV}\) equals \(5.931 \cdot 10^7 \text{ cm/sec}\). The deuteron velocity corresponding to such energy will be assumed to be 3680 times less, and in our units it will be \(5.372 \cdot 10^{-7}\) (if \(c = 3 \cdot 10^{10} \text{ cm/sec}\)). Then the deuteron movement towards the nucleus is described by the equation
\[ \ddot{x} = -\frac{2q^2}{x^2} \cos^2 \left( \frac{1}{2} (t + \tau_0) \right) \dot{x}^2 + x \dot{x} + \phi_0, \]  
(5)

where the parameter \( \tau_0 \) is defined under the condition that the argument of cosine equals \( \phi_0 \) for \( t = 0, x = x_0, \dot{x} = \dot{x}_0 \) (thus \( \tau_0 = -(2x_0)/\dot{x}_0 \)), and this parameter may be considered as the initial moment of so called local time.

We are particularly interested in solutions of (5) under very small deviation \( \varepsilon \) from the phase \( \phi_0 = \frac{\pi}{2} + \varepsilon \) and rewrite (5) in the following form:

\[ \ddot{x} = -\frac{a}{x^2} \sin^2 \left( \frac{1}{2} (t + \tau_0) \right) \dot{x}^2 + x \dot{x} + \varepsilon, \]  
(6)

where \( a = 0.0144967 \). Let the initial \( x_0 \) be equal \(-500000\) of our length units (i.e. approximately \( 5 \times 10^{-9} \text{ cm} \)) and the initial deuteron velocity \( v_0 \) be equal to the velocity \( v_{00} \) corresponding to the deuteron energy of 1 eV or less. But it turned out that the precision of numerical integration of this equation under such initial conditions and under values \( |\varepsilon| = 10^{-15} \) and less is small and besides the interval of the integration must be very large. That is why this equation also had to be transformed by passing to “slow” time \( \tau = |\varepsilon| t \) to the equation with respect to the variable \( w = \left( \frac{dx}{d\tau} \right)^2 \) as a function of \( x \):

\[ \frac{dw}{dx} = -\frac{2a}{x^2} \sin^2 \left[ \frac{1}{2} (\tau + \tau_0) w + x \sqrt{w} \pm 1 \right], \]  
(7)

where \( \tau_0 = -(2x_0)/\sqrt{w(x_0)} \) and \( +1 \) if \( \varepsilon > 0 \), and \( -1 \) if \( \varepsilon < 0 \). It must be added also the equation for \( \tau \) as a function of \( x \):

\[ \frac{d\tau}{dx} = \frac{1}{\sqrt{w}}. \]  
(8)

The system of equations (7,8) is, so to say, a “model” system describing fairly accurately (from viewpoint of quantities data) the deuteron movement under all values of \( |\varepsilon| \) from \( 10^{-24} \) to \( 10^{-6} \).

The numerical integration of this system was carried out under different values of \( \varepsilon \) and under following initial conditions:

\[ w(x_0) = 2.103 \pi(x_0) = 0, x_0 = -500000, \dot{x}_0 = 6895738 \]  
(9)

It may be noted that the initial deuteron velocity \( v_0 \) equals 1.450172 (following the relation \( \dot{x}_0 = |\varepsilon| \sqrt{w(x_0)} \)) for given initial \( w(x_0) \) and for \( |\varepsilon| = 10^{-7} \), i.e. such velocity is approximately 3.7 times less than velocity \( v_{00} \) corresponding the deuteron energy of 1 eV. If \( |\varepsilon| = 10^{-6} \) then the velocity \( v_0 \) is approximately 2.7 times greater than velocity \( v_{00} \).
It turned out that the numerical tables for values of $w, \tau$ obtained under different values of $\varepsilon < 0$ in the interval $(-10^{-24}, -10^{-6})$ don’t differ essentially from each other. The following table is true up to three-four significant figures for $\tau$ and $\dot{x}/|\varepsilon| = \sqrt{w}$:

| $x$   | $\tau$ | $\dot{x}/|\varepsilon|$ |
|-------|--------|---------------------|
| -500 000 | 0      | 1.450               |
| -50 000  | $1.426 \cdot 10^6$ | 0.0493             |
| -500     | $1.002 \cdot 10^7$ | 0.000489          |
| -200     | $1.067 \cdot 10^7$ | 0.000440          |
| -100     | $1.090 \cdot 10^7$ | 0.000425          |
| -80      | $1.100 \cdot 10^7$ | 0.000423          |

If reducing the table values of $x$ to centimeters, we obtain the following corresponding approximate values:

$5 \cdot 10^{-9}, 5 \cdot 10^{-10}, 5 \cdot 10^{-12}, 2 \cdot 10^{-12}, 10^{-12}, 0.8 \cdot 10^{-12} \text{ cm}$

The time interval $\Delta T$, in which the deuteron reaches the critical distance $10^{-12} \text{ cm}$ from the center is $1.090 \cdot 10^7 / |\varepsilon|$ of our time units or $(1.090 \cdot 10^7 / |\varepsilon|) \cdot 3.502 \cdot 10^{-25}$ seconds. If nuclear forces are not taken into account then the deuteron may approach the distance less $10^{-12} \text{ cm}$ We present here for illustration the table, where the initial deuteron velocities $v_0$ in velocities shares $v_{00}$ and the corresponding time intervals $\Delta T$ (in seconds) for different values of $\varepsilon$ are listed.

<table>
<thead>
<tr>
<th>$\varepsilon$</th>
<th>$\frac{v_0}{v_{00}}$</th>
<th>$\Delta T$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-10^{-6}$</td>
<td>2.7</td>
<td>$3.82 \cdot 10^{-12}$</td>
</tr>
<tr>
<td>$-10^{-7}$</td>
<td>0.27</td>
<td>$3.82 \cdot 10^{-11}$</td>
</tr>
<tr>
<td>$-10^{-22}$</td>
<td>$0.27 \cdot 10^{-15}$</td>
<td>$3.82 \cdot 10^4$ ($\approx 10.6$ hours)</td>
</tr>
<tr>
<td>$-10^{-23}$</td>
<td>$0.27 \cdot 10^{-16}$</td>
<td>$3.82 \cdot 10^5$ ($\approx 106$ hours).</td>
</tr>
</tbody>
</table>

Let us note that the given data change essentially under positive values of $\varepsilon$ ($10^{-6}, 10^{-7}$, etc.) There is some asymmetry of solutions behavior under negative and positive values of $\varepsilon$. The calculations show the minimal distance $|x|_{\min}$ more than 500 of our lengths units even for relative big initial $w(x_0) = 10000$. Thus, if we limit ourselves to the condition that the deuteron energy is not over $(0.27)^2 \text{ eV}$ at a distance of $5 \cdot 10^{-9}$ cm from the central nucleus, and the whole process of deuteron movement towards the nucleus does not exceeds approximately 10.6 hours, then the interval $(\frac{\pi}{2} - 10^{-7}, \frac{\pi}{2} - 10^{-22})$ is approximately the sought phase hole in the whole interval $(0, \pi)$ of phase change $\varphi_0$ in eq. (5).
If many deuterons with energy not more than \((0.27)^2 \text{ eV}\) at the distance \(5 \times 10^{-9} \text{ cm}\) from the nucleus are equally distributed along their phases \(\varphi_0\), the ratio of the length of this hole to \(\pi\), equaling approximately \(0.3 \times 10^{-7}\), is equal to the share (or the respective percentage of \(0.3 \times 10^{-5}\)) of deuterons overcoming the Coulomb barrier.

The above figures express at least the order of probability of the LERN occurrence, and this order is absolutely incompatible with the figures in the standard quantum mechanics mentioned above. Let us note once again that a one-dimensional problem was solved, and in case of an accurate analysis (not zero sighting distance will be taking into account) this probability will be lower. Let us also pay attention to the large time intervals \(\Delta T\) calculated if \(|e|\) is very small. It explains well the effect (observed by many researchers) of continuation of cold fusion reactions even many hours after disconnection of the voltage in the electrolytic cells. This effect was named even “life after death”.

As for the analysis of the deuteron movement with the help of the autonomous equation, the calculations lead to initial velocities \(v_0\), exceeding the above mentioned numbers, although the general motion picture is the same. But the autonomous equation is interesting, because in the area of those values \(x, \dot{x}\), under which the product \(x \dot{x}\) has a small modulo, it is possible to replace \(\sin(x \dot{x})\) with \(x \dot{x}\), and consider under \(\varepsilon = 0\) the following equation (describing the deuteron motion from initial point \(x_0 > 0\) to the center)

\[
\ddot{x} = a \frac{(x \dot{x})^2}{x^2} = a \dot{x}^2
\]

This equation has a very simple analytical solution. Without giving very simple calculations, we will present the final formulas.

Let us take the following initial conditions:

\[
x(0) = x_0 > 0, \quad \dot{x}(0) = -v_0 < 0
\]

Then

\[
\dot{x}(t) = -\frac{v_0}{1 + av_0 t}, \quad x(t) = x_0 - \frac{1}{a} \ln(1 + av_0 t).
\]

It follows from these formulas that the velocity of a particle moving in accordance with the initial equation never turns to zero, and under

\[
t = t_* = \frac{\exp(ax_0) - 1}{av_0}
\]

\(x(t_*) = 0\), i.e. the particle reaches the center of the nucleus, its velocity at this moment being

\[
\dot{x}(t_*) = \frac{-v_0}{1 + av_0 t_*} = -v_0 \exp(-ax_0),
\]

so that it passes through the nucleus and moves further!

For example, let \(a=0.0144967\), \(x_0 = 1000\) (\(\approx 10^{-11} \text{ cm}\)), \(\dot{x}(0) = 5.37 \times 10^{-10}\) (\(\approx 16 \text{ cm/s}\)). Under such initial data, the product \(x \dot{x} = -0.0000537\), so it is quite possible to replace \(\sin(x \dot{x})\) with \(x \dot{x}\). In this case,
\[ t_* \approx 2.3 \cdot 10^7 \ (\approx 8 \cdot 10^{-18} \text{ sec}), \]
\[ \dot{x}(t_*) \approx -29.9 \cdot 10^{-17} \ (\approx 9 \cdot 10^{-6} \text{ cm/sec}) \]

These figures fit well into the reasonable framework, so the autonomous model can also be of use for the movement analysis in the problem under review. The phenomenon of particle passage through the Coulomb potential accounts very well for the existence of pendulum orbits in the Bohr-Sommerfeld model, when in states 1s, 2s, 3s etc. the electron passes through the nucleus. Such states in the strict theory and experiment have no impulse, so in the Bohr-Sommerfeld model they were discarded as absurd. Now they have a right to exist. Further, the experimental data for angular distribution of non-elastic scattering by nuclear reactions (including reactions with heavy ions) reveal the big amplitude of the scattering forward. It is impossible to explain such effect by the formation of intermediate nuclei but it may be explained from the viewpoint of our UQT.

5. Harmonics oscillator at Unitary Quantum Theory and Energy Generation

Let us examine two variants of equations (3,4) for parabolic potential \( U \sim x^2 \) in the scalar case:

\[ \ddot{x} = -2qx \cos^2(-x\dot{x} + \phi) \quad (10) \]

(autonomous equation) and

\[ \ddot{x} = -2qx \cos^2\left(\frac{1}{2} \dot{x}^2 t - x\dot{x} + \phi\right) \quad (11) \]

(non-autonomous equation),

where \( q \) is the constant part of particle’s oscillating charge and \( \phi \) is the initial phase, that may be represented as \( \phi = \pi/2 + \varepsilon \), where \( \varepsilon \) - phase deviation from \( \pi/2 \). As far as cosine is squared, it is quite enough to examine different values of \( \phi \) and \( \varepsilon \) within intervals from 0 to \( \pi \) or from \( -\pi/2 \) to \( \pi/2 \).

The character of the particle motion to be described by these equations essentially depends just on \( \varepsilon \). So we substitute equations (10), (11) for the following:

\[ \ddot{x} = -2qx \sin^2(-x\dot{x} + \varepsilon), \quad (12) \]
\[ \ddot{x} = -2qx \sin^2\left(\frac{1}{2} \dot{x}^2 t - x\dot{x} + \varepsilon\right). \quad (13) \]

The numerical integration of these equations [21-29] yielded four types of solutions:
Inter
damped oscillations with amplitude, tending to zero; meanwhile particles sometimes assume
a “phantom” state; in that case their wave packets are spread all over Universe;
irregular oscillations, remaining constant over a long period of time, thus yielding a quasi-stable
situation;
oscillations with monotone increasing amplitude. In some cases these oscillations may abruptly
enter a trajectory towards infinity; meanwhile cosine argument and the particle’s charge
approach zero. It may be said that in that case the particle abruptly assumes a “phantom” state;
the particle almost immediately enters an escape trajectory and rapidly approaches the
“phantom” state without any preliminary oscillations (it can be said without “preliminary
doubts”).

In summary, only four variants of particle motion are possible: energy increase or decrease,
stable and with vanishing particle (transformation into the “phantom” state).

These solutions have been reported for the first time by one of the authors (L.S.) at the
conference ICCF5 taking place in Monte-Carlo [26] and published in [21-29], and called: «maternity
home», «crematorium», stable and “ghostly”. The first three solutions correspond, in general, to Fig. 4.
The solution passing into “phantom” state has analogous to solutions of Shroedinger’s equation
containing Hermite functions, because the exponential “tails” of the wave function exist always out of
parabolic well.

The standard quantum theory carefully avoids the question of conservation laws for single
events at small energies. Usually that question either does not being discussed at all, or there are said
some words that quantum theory does not describe single events at all. But these words are wrong,
because the standard quantum theory describes, in fact, single events, but is able to foreseen only the
probability of that or other result. It is evident that at that case there are no conservation laws for single
events at all. These laws appear only after averaging over a large ensemble of events [30]. As the
matter of fact it can be easily shown that classical mechanics is obtained from quantum one after
summation over a large number of particles. And for a quite large mass the length of de Broglie wave
becomes many times less than body dimensions, and then we cannot talk about any quantum-wave
characteristics any more.

Fig. 4.

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6. Conclusion

Numerous experiments with the LERN (including the latest of Andrea Rossi - Italy) have shown that nuclear reactions do exist but the nuclear reactions products by themselves are not enough for the explanation of huge amount of heat being produced. It is the responsibility of the UQT solutions “Maternity home” [3, 4, 21]. So it looks like catalysis mechanism described [3, 4, 22]. Besides all the equation with oscillating charge (3, 4, 5, 21-29) is quite good in describing the wave properties of the particle. We predict that experiments on the diffraction reflection of electrons from the lattice (classical experiments of Davisson-Germer) can be simulated by supercomputer, but authors do not have such possibility.

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